

**What is Claimed:**

1. A method of synthesizing pulses for a given selective excitation profile in a system including at least one of a two-level quantum system and a subsystem described by the spin domain Bloch equation, comprising the steps of:
  - determining frequency envelopes to produce the given selective excitation profile;
  - producing a plurality of solutions via an inverse scattering transform producing said envelopes, including identifying and producing a minimum energy solution to said inverse scattering transform for pulses that generate the given selective excitation profile; and
  - generating pulses corresponding to any of the plurality of solutions to produce the envelopes that produce said given selective excitation profile.
2. A method as in claim 1, wherein said system includes one of a quantum computer system and a spintronics system.
3. A method as in claim 1, wherein said system is a magnetic resonance imaging system, said pulses comprise radiofrequency (RF) pulses, said frequency envelopes comprises RF envelopes, and said selective excitation profile comprises an arbitrary magnetization profile.
4. A method as in claim 3, wherein said arbitrary magnetization profile is an arbitrary unit 3 vector valued function of a single real variable.
5. The method of claim 3, wherein said solutions producing step comprises the step of using a reflection coefficient corresponding to a hard pulse approximation to approximate an ideal reflection coefficient that is determined by said given magnetization profile.
6. The method of claim 1, wherein said solutions producing step comprises the step of specifying bound states and norming constants for reduced scattering data subject to constraints on the energy to be used in said inverse scattering transform.
7. The method of claim 1, comprising the additional step of increasing the energy of the RF envelope in order to reduce rephasing time while maintaining a constraint on the energy to be used in said inverse scattering transform.

8. The method of claim 1, comprising the additional step of generating a softened pulse approximation to said RF envelope from the RF pulses generated in said generating step.
9. A method of synthesizing pulses that produce a given selective excitation profile in a system including at least one of a two-level quantum system and a subsystem described by the spin domain Bloch equation, comprising the steps of:
  - selecting an approximation to the given selective excitation profile;
  - selecting auxiliary data including bound states and norming constants for said given selective excitation profile;
  - constructing left and right Marchenko equations from the selected bound states and norming constants;
  - solving the left and right Marchenko equations to provide a pulse envelope that produces said given selective excitation profile; and
  - generating pulses to produce said pulse envelope.
10. The method of claim 9, wherein the step of solving the left and right Marchenko equations includes the step of using a discrete inverse scattering transform (DIST) to obtain pulse envelopes from the given selective excitation profile and auxiliary data.
11. The method of claim 10, wherein the step of using the DIST to obtain pulse envelopes includes the steps of:
  - using a hard pulse approximation to approximate a reflection coefficient that is determined by said given selective excitation profile by an exponential polynomial;
  - if there is auxiliary data, encoding the auxiliary data as further exponential polynomials;
  - constructing left and right discrete Marchenko equations from the encoded auxiliary data;
  - using a recursive algorithm to solve the left and right discrete Marchenko equations;
  - and
  - constructing a pulse envelope from solutions to the left and right discrete Marchenko equations.

12. The method of claim 9, wherein said pulse envelope generating step comprises the step of generating a minimum energy pulse that produces the given selective excitation profile.
13. The method of claim 9, wherein said pulse envelope generating step comprises the step of generating a  $\theta$ - flip for offset frequencies lying in a given frequency band for magnetic resonance imaging.
14. The method of claim 9, wherein said pulse envelope generating step comprises the step of generating a  $\theta$ - flip for offset frequencies lying in a collection of frequency bands for magnetic resonance imaging.
15. The method of claim 9, wherein said pulse envelope generating step comprises the step of generating inversion and refocusing pulses for magnetic resonance imaging.
16. The method of claim 9, wherein said pulse envelope generating step comprises the step of generating minimum and maximum phase pulses for magnetic resonance imaging.
17. The method of claim 9, wherein said pulse envelope generating step comprises the step of generating low energy self refocused pulses for magnetic resonance imaging.
18. The method of claim 9, wherein said pulse envelope generating step comprises the step of generating pulses where a flip angle profile is an arbitrarily specified function and the phase is arbitrarily specified for use in magnetic resonance imaging.
19. A method of generating a desired frequency dependent excitation in a system including at least one of a two-level quantum system and a subsystem described by the spin domain Bloch equation using selective pulses for a given selective excitation profile corresponding to said desired frequency dependent excitation, comprising the steps of:
  - determining frequency envelopes to produce the given selective excitation profile;
  - producing a plurality of solutions via an inverse scattering transform producing said envelopes, including identifying and producing a minimum energy solution to said inverse scattering transform for pulses that generate the given selective excitation profile;
  - generating pulses corresponding to any of the plurality of solutions to produce the

envelopes that produce said given selective excitation profile; and

applying the generated pulses to said system to obtain said desired frequency dependent excitation.

20. A method as in claim 19, wherein said system is a magnetic resonance imaging system, said pulses comprise radiofrequency (RF) pulses, said frequency envelopes comprises RF envelopes, and said selective excitation profile comprises an arbitrary magnetization profile.

21. The method of claim 19, wherein said applying step comprises the steps of generating a softened pulse approximation to said envelopes from the pulses generated in said generating step and applying said softened pulse approximation to said system.

22. A method of generating a desired frequency dependent excitation in a system including at least one of a two-level quantum system and a subsystem described by the spin domain Bloch equation using selective pulses for a given selective excitation profile corresponding to said desired frequency dependent excitation, comprising the steps of:

selecting an approximation to the given selective excitation profile;

selecting auxiliary data including bound states and norming constants for said given selective excitation profile;

constructing left and right Marchenko equations from the selected bound states and norming constants;

solving the left and right Marchenko equations to provide a pulse envelope that produces said given selective excitation profile;

generating pulses to produce said pulse envelope; and

applying the generated pulses to said system to obtain the desired frequency dependent excitation.

23. The method of claim 22, wherein said pulse envelope generating step comprises the step of generating a minimum energy pulse that produces the given selective excitation profile.

24. The method of claim 22, wherein said pulse envelope generating step comprises the step of generating a  $\theta$ - flip for offset frequencies lying in a given frequency band for magnetic resonance imaging.
25. The method of claim 22, wherein said pulse envelope generating step comprises the step of generating a  $\theta$ - flip for offset frequencies lying in a collection of frequency bands for magnetic resonance imaging.
26. The method of claim 22, wherein said pulse envelope generating step comprises the step of generating inversion and refocusing pulses for magnetic resonance imaging.
27. The method of claim 22, wherein said pulse envelope generating step comprises the step of generating minimum and maximum phase pulses for magnetic resonance imaging.
28. The method of claim 22, wherein said pulse envelope generating step comprises the step of generating low energy self refocused pulses for magnetic resonance imaging.
29. The method of claim 22, wherein said pulse envelope generating step comprises the step of generating pulses where a flip angle profile is an arbitrarily specified function and the phase is arbitrarily specified for use in magnetic resonance imaging.